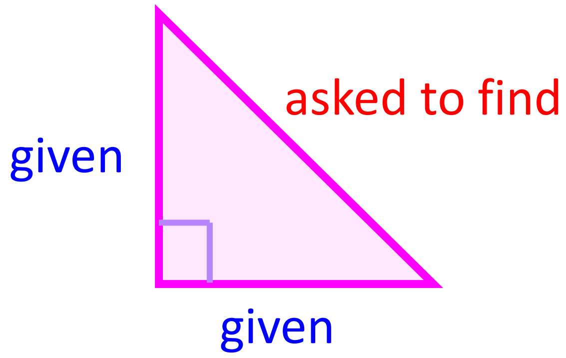
# Pythagoras

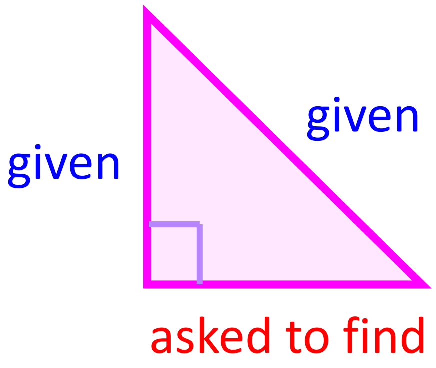


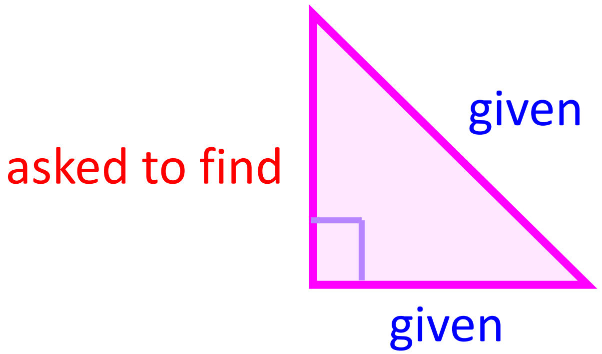
## How Do We Know When To Use Pythagoras?

- **Criteria 1**: If we have a **right angled** triangle
- **Criteria 2:** If we are given the length of TWO sides of a triangle, and we want to find the length of the THIRD side i.e. when we have a triangle and **ONLY sides** are **involved.** (this is in contrast with the topic SOHCAHTOA which you'll learn later where sides AND angles are involved, not only sides)

So, we use Pythagoras if we have one of the following 3 scenarios :



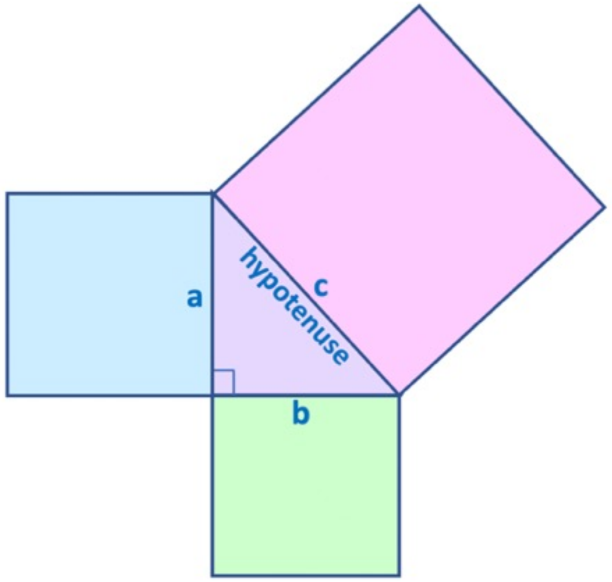



# Background Information (optional reading):

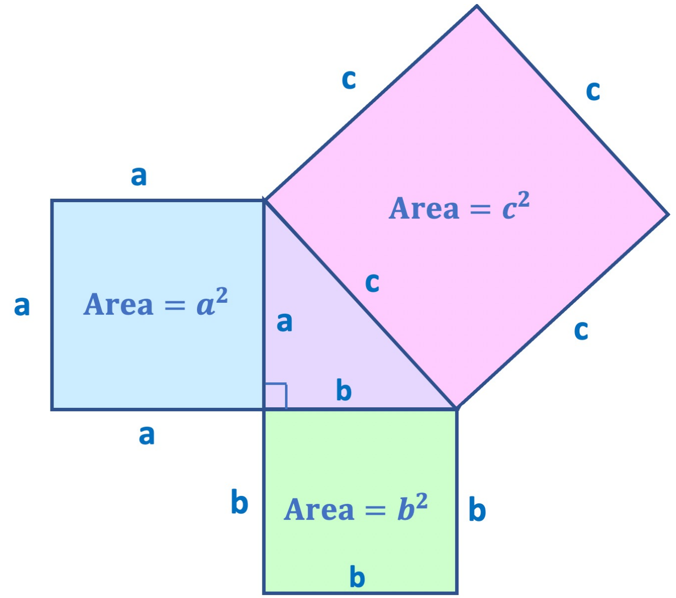


If we have a **right-angled** triangle (the purple one below on the left) and we form squares on each side of the triangle (blue, green and pink squares)

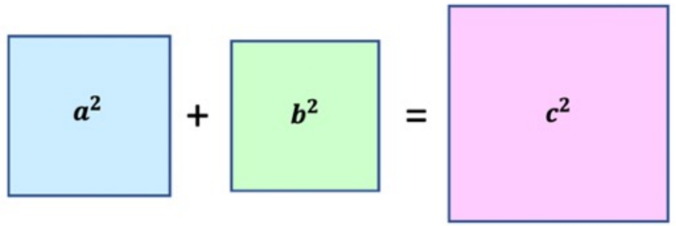



Now we consider the areas of the squares

The green square has area $a \times a = a^2$
The blue square has area $b \times b = b^2$
The pink square has area $c \times c = c^2$




The pink square happens to have the same area as the two other squares (blue and green) put together . See the next page if you'd like to see how.

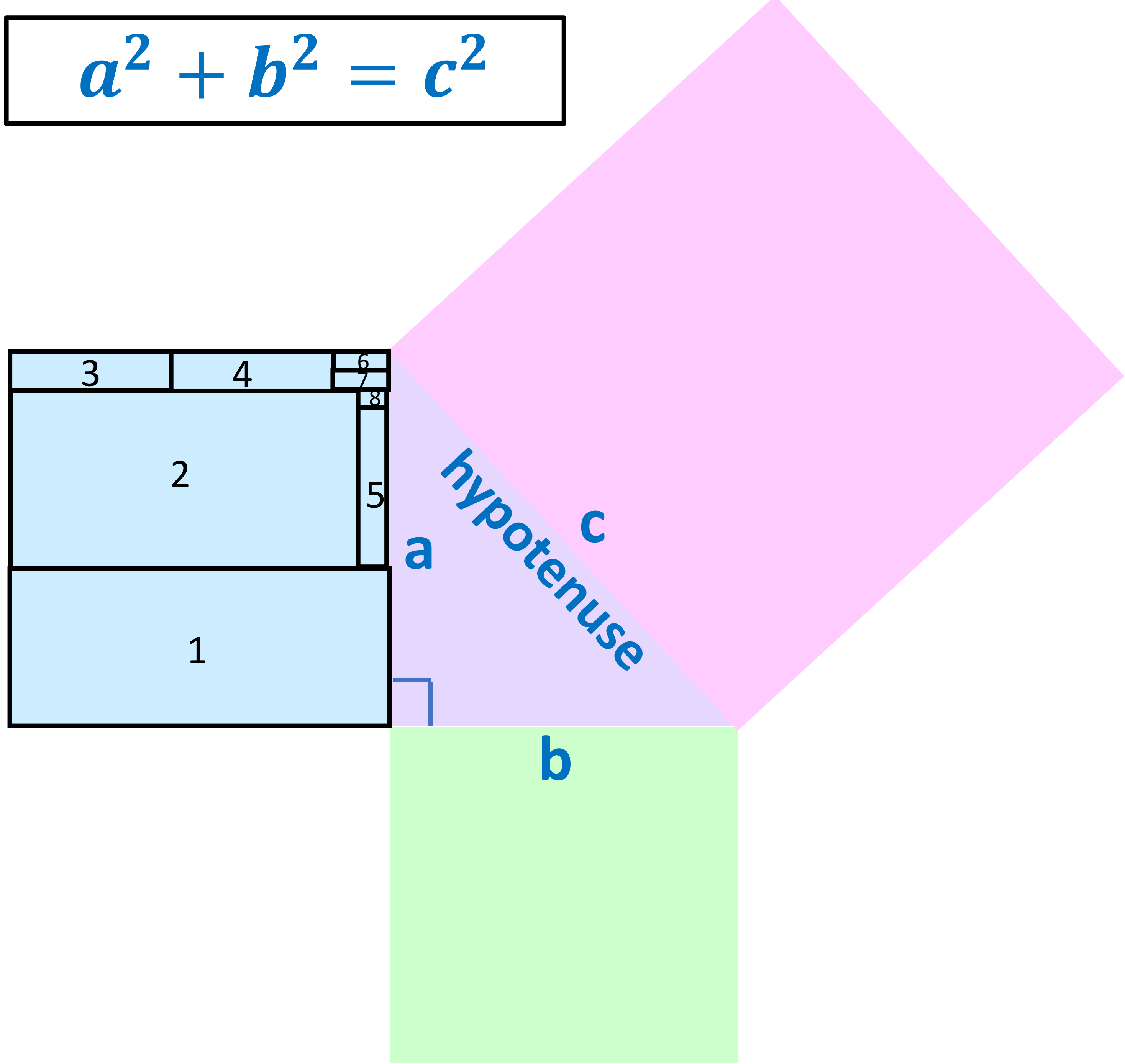


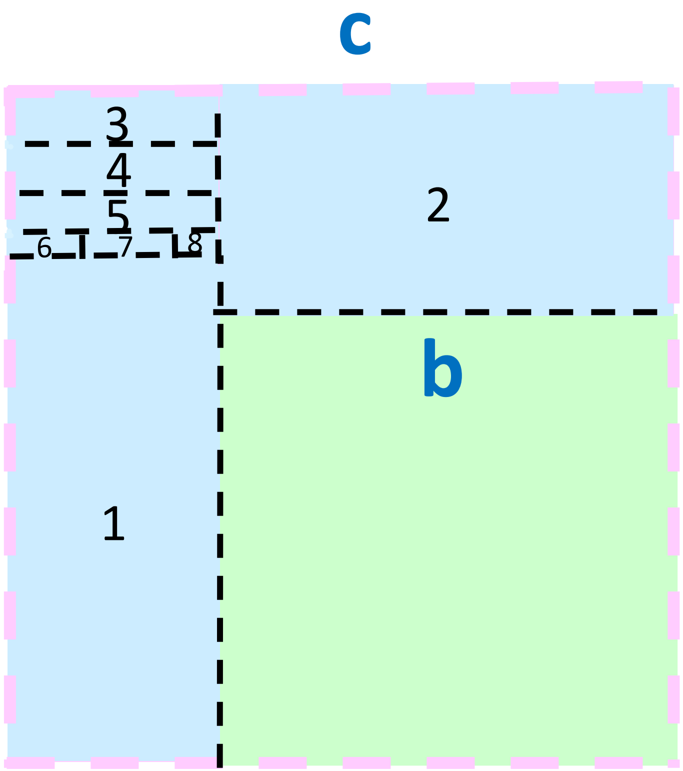

Hence, we can say the area of the square on the hypotenuse is equal to the sum of the areas of the squares (area of blue square plus area of green square) on the other two sides. This is known as the Pythagorean theorem which just says $a^2 + b^2 = c^2$ where $a$ and $b$ are the shorter sides and $c$ is the longest side of the triangle, called the **hypotenuse** (the hypotenuse is the side OPPOSITE the right angle).

We can write the formula in a simpler way as :

$$side\ 1^2 + side\ 2^2 = hypotenuse^2$$

$$a^2 + b^2 = c^2$$




The green square slots in and the blue square has been cut up.
Note: we could have slotted the blue square in and cut up the green square

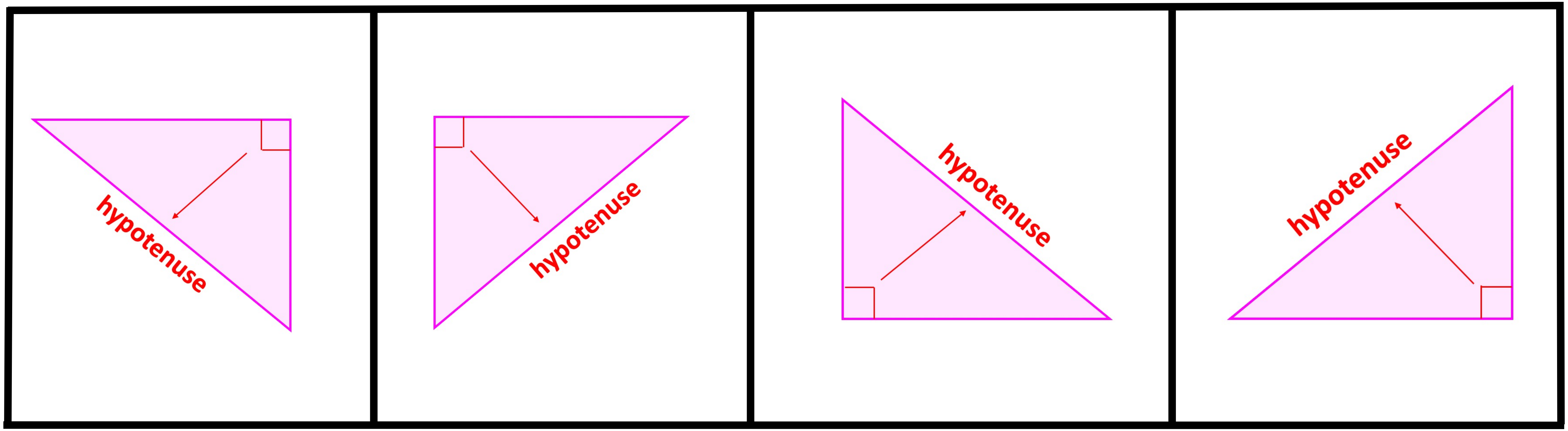
# Long Method

Note: There is a a shorter method on page 8 which you may prefer to go to

# Step 1 :



Locate the hypotenuse.  The hypotenuse is always the side opposite the right angle.

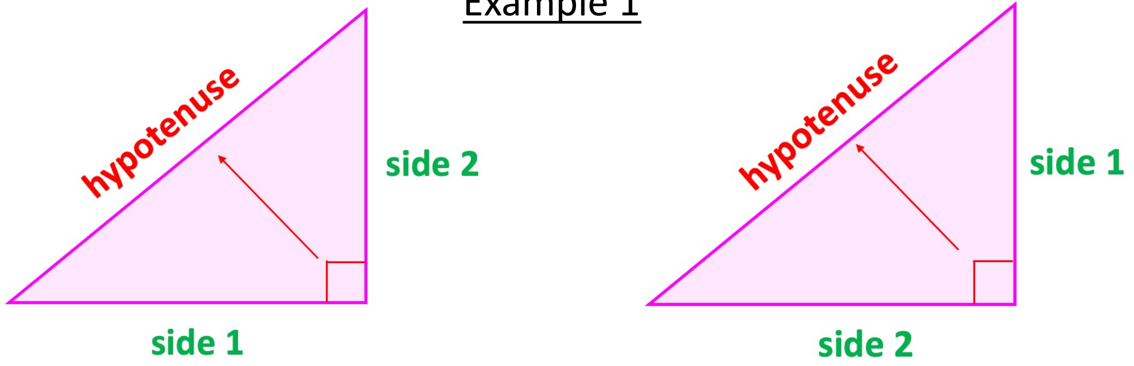


# Step 2 :



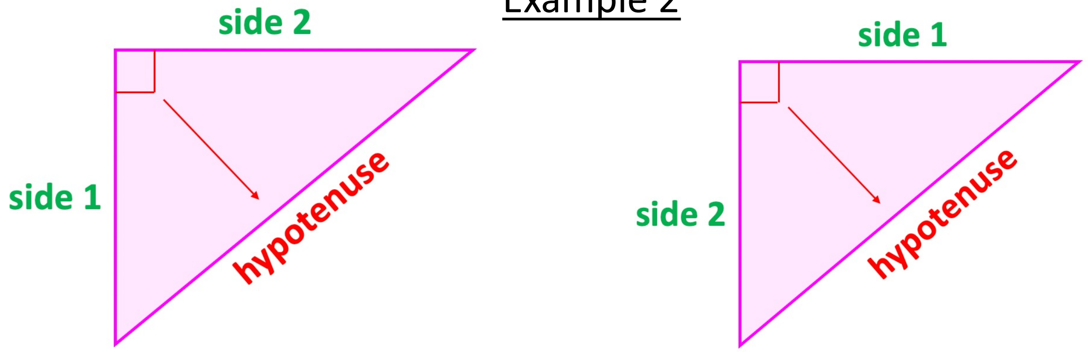
Locate the other 2 sides of the triangle.

Example 1



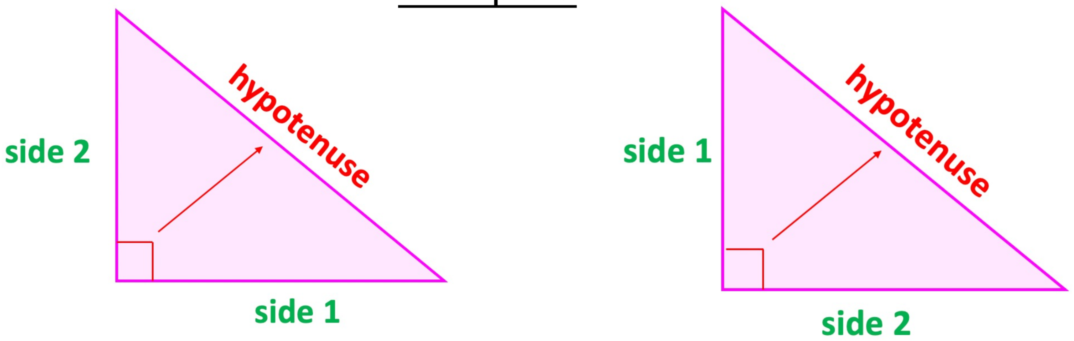

It doesn't matter which sides we call side 1 and side 2

Example 2



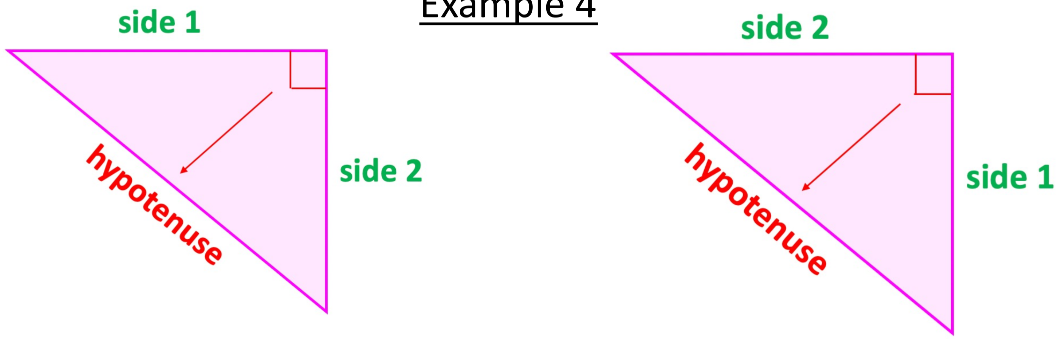

It doesn't matter which sides we call side 1 and side 2

Example 3




It doesn't matter which sides we call side 1 and side 2

Example 4




It doesn't matter which sides we call side 1 and side 2

**Step 3 :** Fill the values from the sides into the formula $side\ 1^2 + side\ 2^2 = hypotenuse^2$

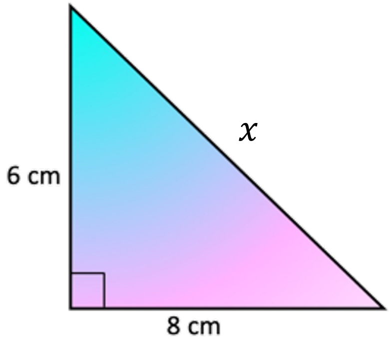
Recall that Pythagoras writes this as $a^2 + b^2 = c^2$

We then use algebra (if necessary) to re-arrange and solve for the unknown side



Let's look at 2 examples:

**Example 1**




We fill into the formula $a^2 + b^2 = c^2$. It doesn't matter which side we call $a$ or $b$ (see 2 ways below)

**Way 1:**

$6^2 + 8^2 = x^2$

Simplify

$36 + 64 = x^2$

$100 = x^2$

This is the same as writing

$x^2 = 100$

We can now square root

$x = \sqrt{100}$

$x = 10$

**Way 2:**

$8^2 + 6^2 = x^2$

Simplify

$64 + 36 = x^2$

$100 = x^2$

This is the same as writing

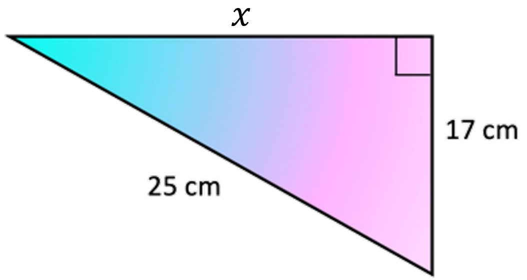
$x^2 = 100$

We can now square root

$x = \sqrt{100}$

$x = 10$

**Example 2**




This question is slightly harder as we are not finding the hypotenuse. We fill into the formula $a^2 + b^2 = c^2$ . It doesn't matter which side we call $a$ or $b$ (see 2 ways below)

**Way 1:**

$17^2 + x^2 = 25^2$

Simplify

$289 + x^2 = 625$

Re-arrange to make $x^2$ the subject

$x^2 = 625 - 289$

$x^2 = 336$

We can now square root

$x = \sqrt{336}$

Using our calculator gives

$x = 18.3$

**Way 2:**

$x^2 + 17^2 = 25^2$

Simplify

$x^2 + 289 = 625$

Re-arrange to make $x^2$ the subject

$x^2 = 625 - 289$

$x^2 = 336$

We can now square root

$x = \sqrt{336}$

Using our calculator gives

$x = 18.3$

# Short Method

# Step 1 :

Ask yourself whether you are **finding** or **given** the hypotenuse (the side opposite the right angle)

# Step 2 : Square both sides and then decide whether to ADD or SUBTRACT and then square root after

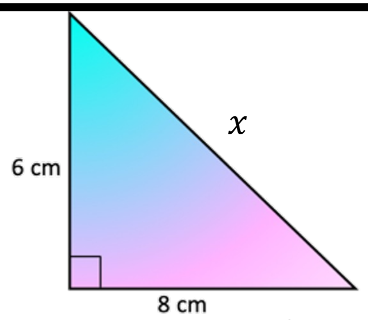
If **FINDING** the hypotenuse: Square the sides and ADD. Square root after.

If **GIVEN** the hypotenuse: Square the sides and SUBTRACT. Square root after

Note: you MUST subtract the smaller number from the bigger number

Let's look at 2 examples:

Example 1

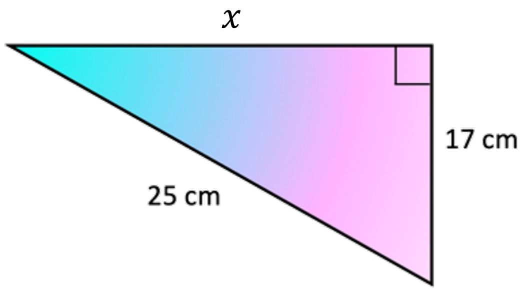



We are finding the hypotenuse (side opposite the right angle)
This means we square both sides, ADD and root

$$\sqrt{6^2 + 8^2}$$
$$= \sqrt{100}$$
$$= 10$$

$$x = 10$$

Example 2




We are given the hypotenuse (side opposite the right angle)
This means we square both sides, SUBTRACT and root

$$\sqrt{25^2 - 17^2}$$
$$= \sqrt{336}$$
$$= 18.3$$

$$x = 18.3$$

# SOHCAHTOA

Just like with Pythagoras, we only use SOHCAHTOA this for right-angled triangles

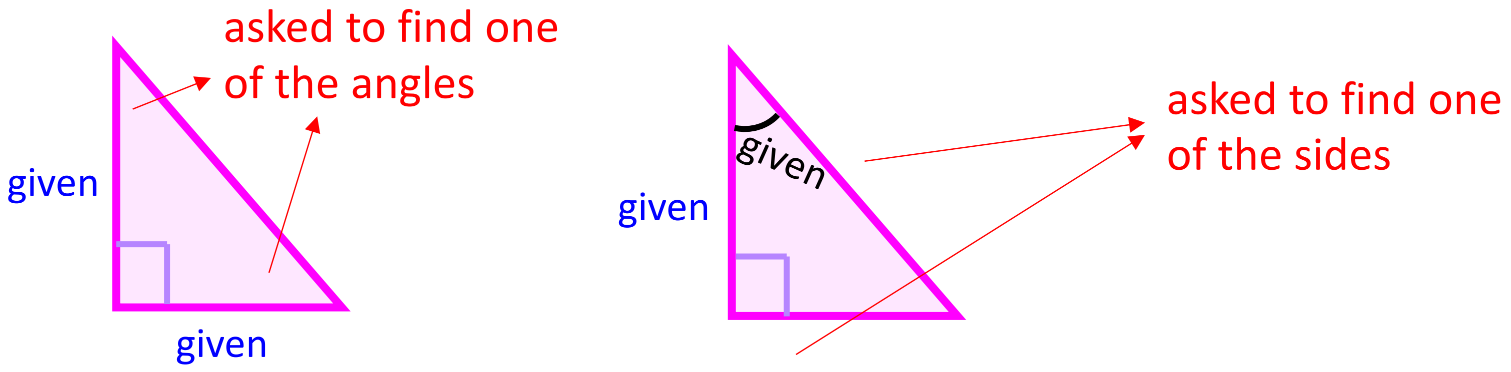
How does SOHCAHTOA differ from Pythagoras?
We use SOHCAHTOA when **sides AND angles** are involved (not just sides like Pythagoras). In other words when we are either:

- ✓ **Given 2 lengths** and want to find an **angle**

or

- ✓ **Given a length** and an **angle** (other than the right angle) and want to find the **length**

Always remember that in **order to be able to use SOHCAHTOA** on a triangle we need to be given **any two lengths** OR given **a length and an angle** (other than the right angle)



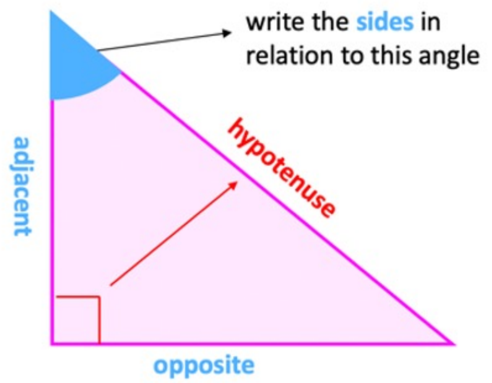
# Short Method

# Step 1 : Label the sides on the triangle as adjacent, opposite and hypotenuse.
(we label the sides in relation to the angle given or the angle we want to find)

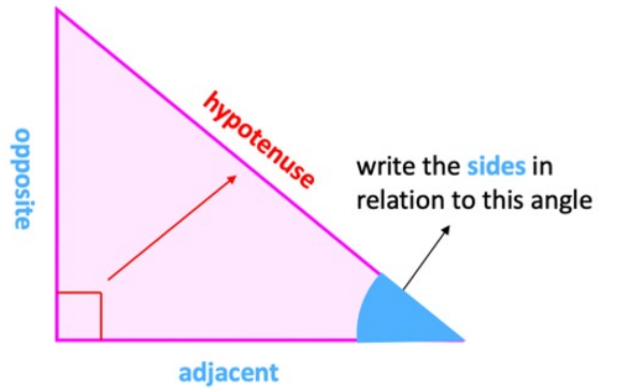
Hypotenuse is always the side opposite to the right angle
Adjacent is always the side right next to the angle (but not the hypotenuse)
Opposite is the side directly opposite the angle

For example: Consider the following 8 scenarios where the blue angle would be the **angle given** OR the **angle we want to find**.

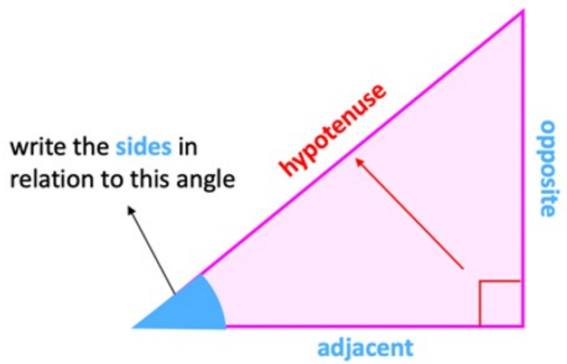
Example 1



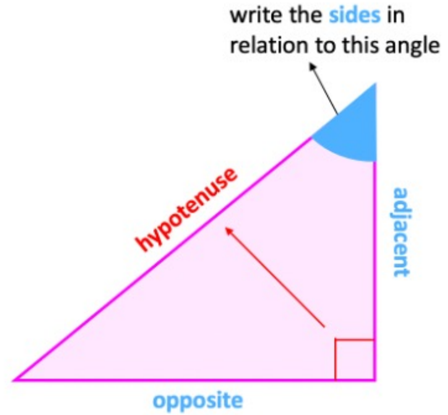

Example 2



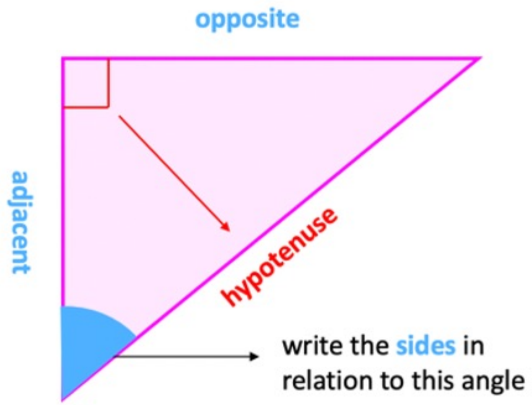

Example 3



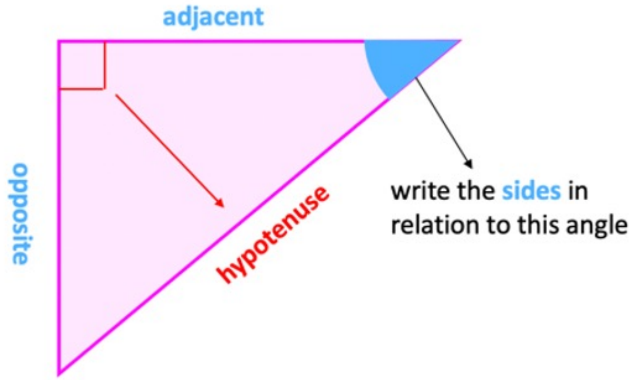

Example 4



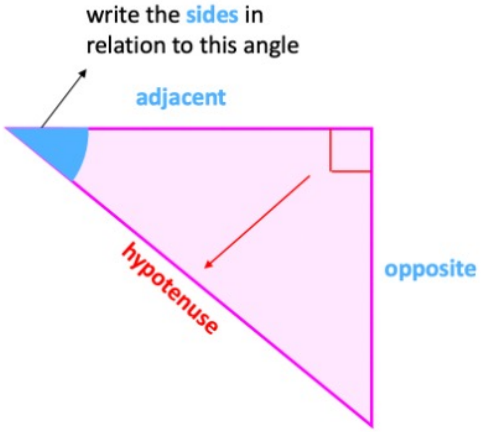

Example 5



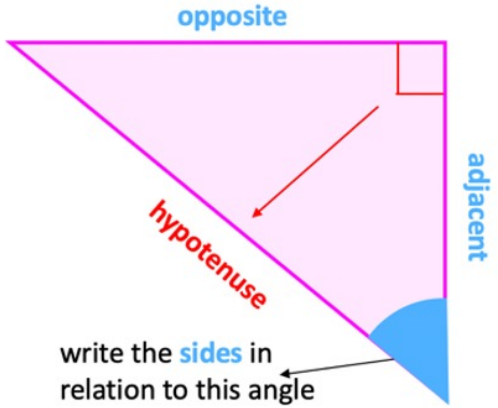

Example 6




Example 7




Example 8



**Step 2 :** Decide whether to use sin/cos/tan based on the sides we care about.
The sides we care about (i.e. sides involved) are either the sides given or the sides asked to find
(the side that isn't given/side we not trying to find is the side we DON'T care about)

- If opposite and hypotenuse sides involved use sin
- If adjacent and hypotenuses sides involved use cos
- If opposite and adjacent sides involved use tan

These can be remembered by SOHCAHTOA

# SOHCAHTOA

sin — If opp and hyp use Sin

cos — If adj and hyp use Cos

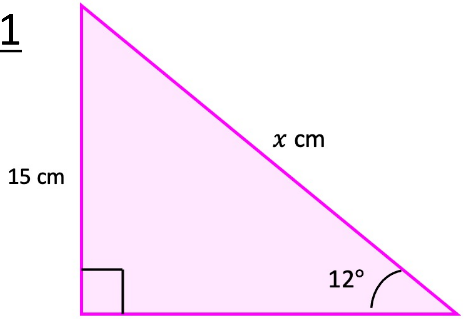
Tan — If opp and adj use Tan

"sock-a-toe-ahhh"

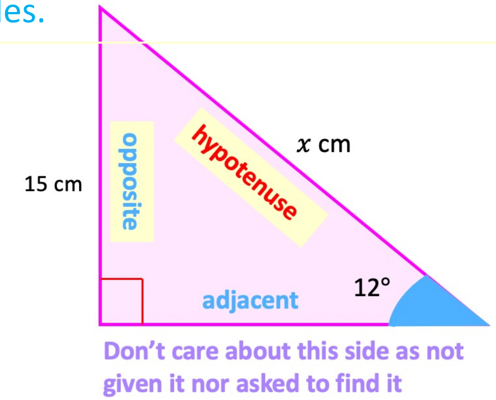
Let’s look at the examples with steps 1 and 2 so far



## Example 1




**Step 1** Label the sides.

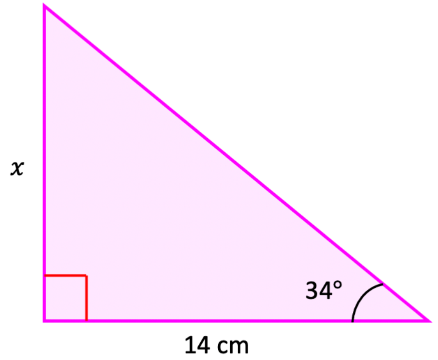



**Step 2** Highlight the sides we care about

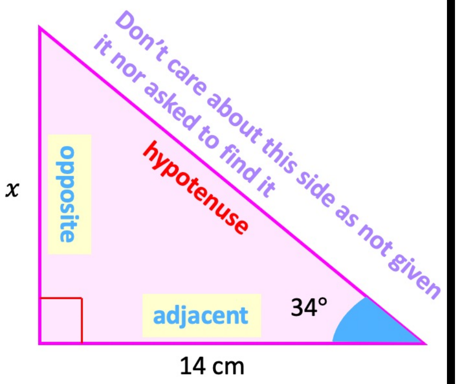
So here we consider **opposite (O)** and **hypotenuse (H)** (we don’t care about the adjacent side since it is not given to us and we are not asked to find it).

Which trig identity uses **O** and **H**?
Consider **SOHCAHTOA**. Remember, **S stands for sin, C stands , for cos and T stands for Tan.**
Here we use **Sin** since we have **O** and **H**.

## Example 2




**Step 1** Label the sides

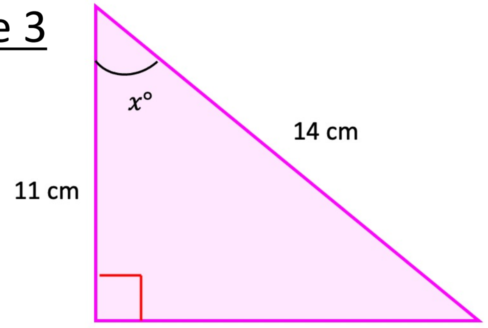



**Step 2** Highlight the sides we care about

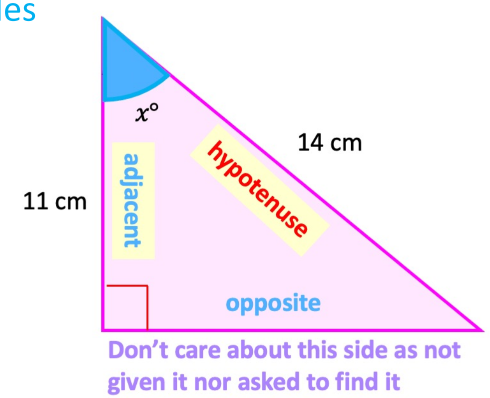
So, here we consider **opposite (O)** and **adjacent (A)** (we don’t care about the hypotenuse since it is not given to us and we are not asked to find it).

Which trig identity uses **O** and **A**?
Consider **SOHCAHTOA**. Remember, **S stands for sin, C stands for cos and T stands for Tan.**
Here we use **Tan** since we have **O** and **A**.

## Example 3




**Step 1** Label the sides



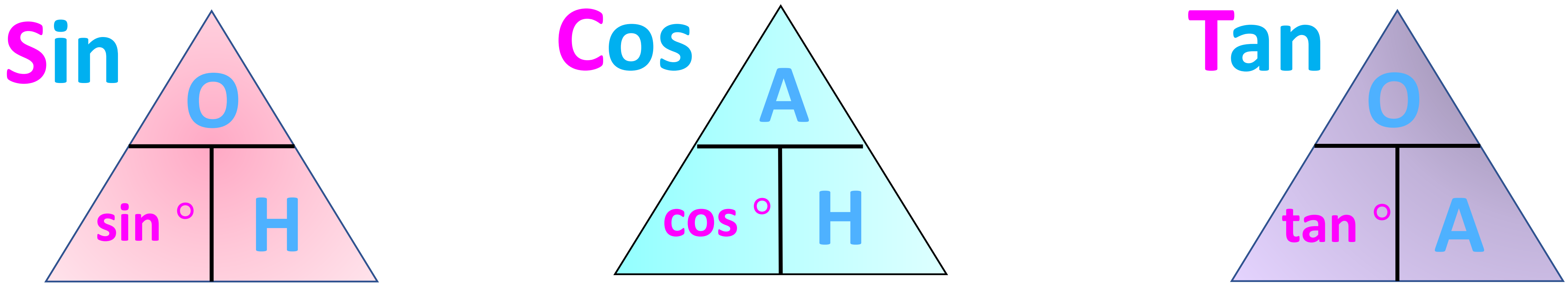

**Step 2** Highlight the sides we care about

So, here we consider **adjacent (A)** and **hypotenuse (H)** (we don’t care about the opposite side since it is not given to us and we are not asked to find it).

Which trig identity uses **A** and **H**?
Consider **SOHCAHTOA**. Remember, **S stands for sin, C stands for cos and T stands for Tan.**
Here we use **Cos** since we have **A** and **H**.

# Step 3 : Option 1 (Pyramid Method - Short Cut Easier Method)

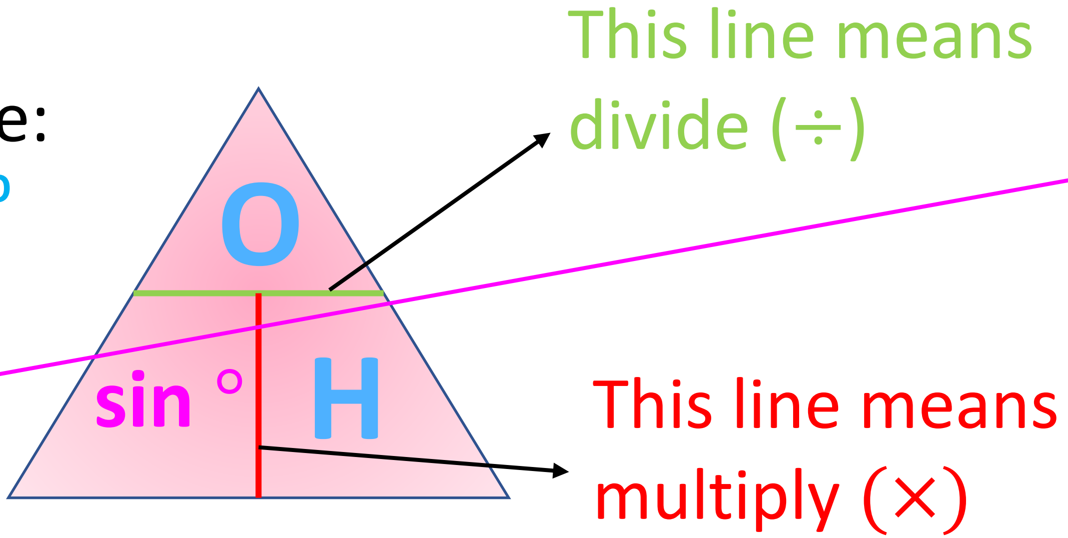
Locate the correct pyramid (based on whether sin, cos or tan was chosen in step 2)




Next, cover up (cross off) what you're trying to find and do the resulting operation ($\div$ *or* $\times$):

Take the sin pyramid for example:

- If we are finding the opposite (O) side, we cross off **O** and do sin (angle) **X H** on the calculator
- If we are finding the hypotenuse (H), we cross off **H**
- and do $\frac{O}{\sin(angle)}$ on the calculator
- If we are finding the angle °, we cross off **sin** (°) and do $\sin^{-1}\left(\frac{O}{H}\right)$ on the calculator




Important: Be careful when finding an angle (i.e. when crossing off sin°). We have a fraction and use the shift button on our calculator

This will make more sense once you see the example on the next page

# Step 3 : Option 2 (Algebraic Method – Longer Harder Method)

Locate the correct formula (based on whether sin, cos or tan was chosen in step 2)

$$sin\ x = \frac{opp}{hyp} \quad , \quad cos\ x = \frac{adj}{hyp} \quad , \quad tan\ x = \frac{opp}{adj}$$

Fill everything you know into the correct template above. Call the unknown any letter you want.

Then use algebra to re-arrange for the unknown.

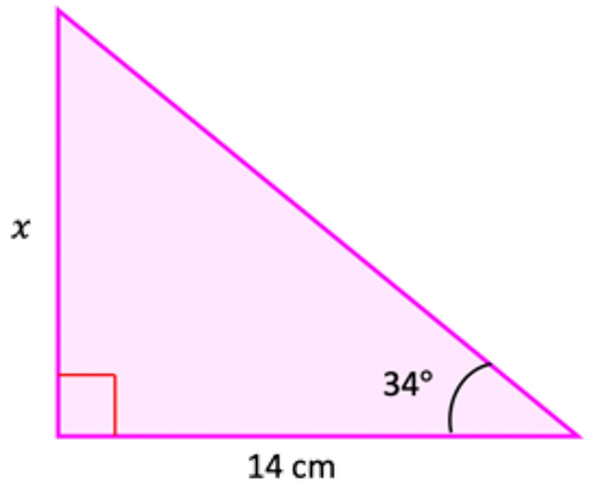
Remember to use the buttons $sin^{-1}, cos^{-1}$ and $tan^{-1}$ **if finding an angle**.
Why? We need this button when re-arranging for the unknown angle. For example $sin\ x = 0.5$ . The only way we can separate the angle $x$ from its trig function is to use the INVERSE of the trig function hence $x = \sin^{-1}\left(\frac{1}{2}\right)$.

Examples are the best way to learn this so don't worry if you don't understand at this point!

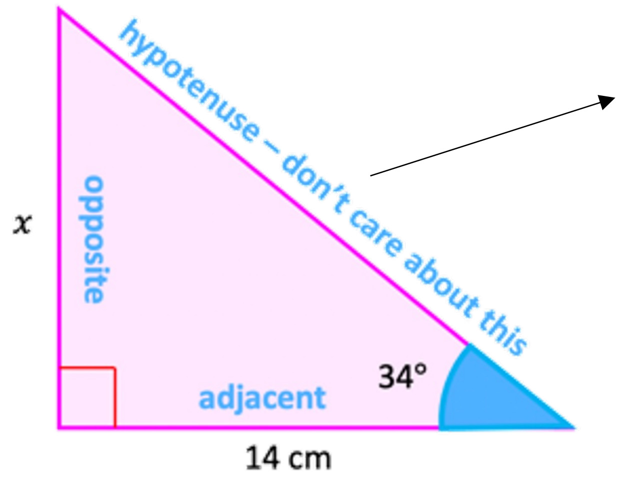
Don't worry if this doesn't make sense! It will after you see the examples below.

# Examples

## Example 1: Finding a side using method 1 (pyramid)



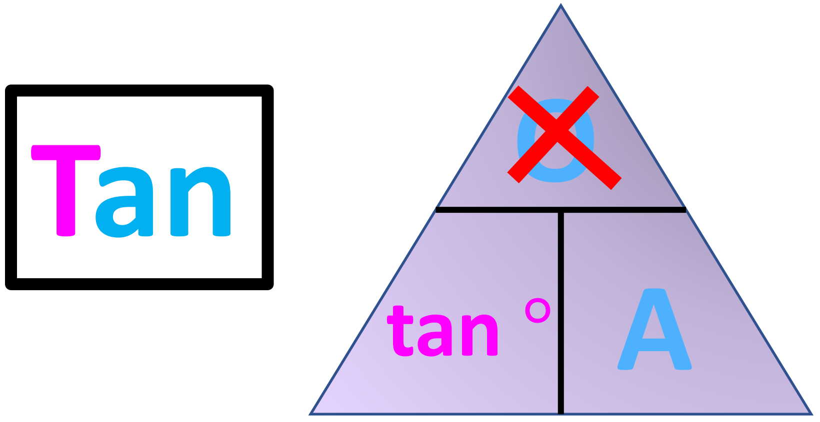

**Step 1:** Label in relation to the angle




Note: we don't care about this since not asked to find it and not given it

**Step 2:** This involves tan since we care about adjacent and opposite, so let's use the Tan triangle. We are trying to find opposite (O), so cover up **O** part.

**Step 3:**

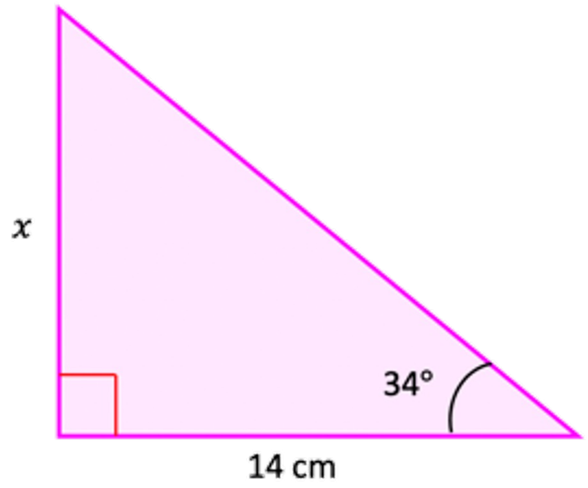


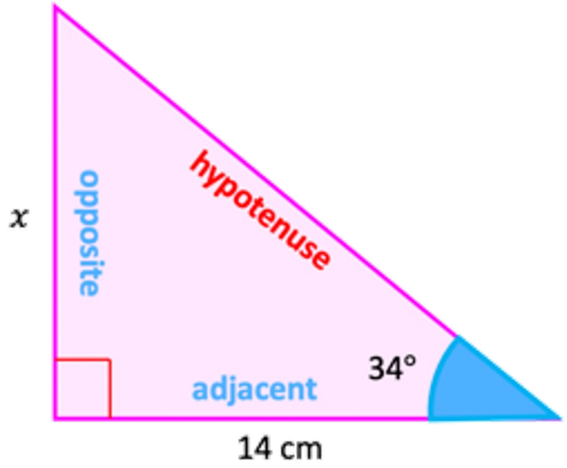

we are left with tan (angle) multiplied by $A$

Remember that T stands for Tan and Tan cannot be written without an angle next to it

$$\Longrightarrow \tan 34 \times 14 = 9.44$$

**Example 1: Finding a side using method 2 (algebraic)**




This involves tan since we care about adjacent and opposite

Fill into formula $\tan x = \frac{opp}{adj}$

$$\tan 34 = \frac{x}{14}$$

Re-arrange for the unknown $x$

Remember $\tan 34$ is just a number, don't let it confuse you

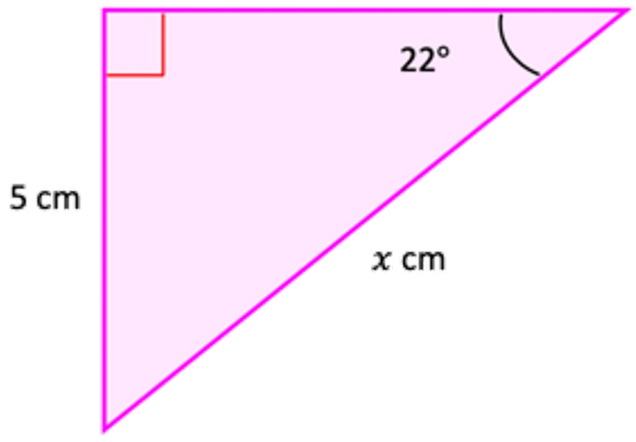
If you had some other equation that was easy like $3 = \frac{x}{5}$, what would you do?

You'd multiply across by 5 to get $x$=15

So do the same here for this example with $\tan 34 = \frac{x}{14}$

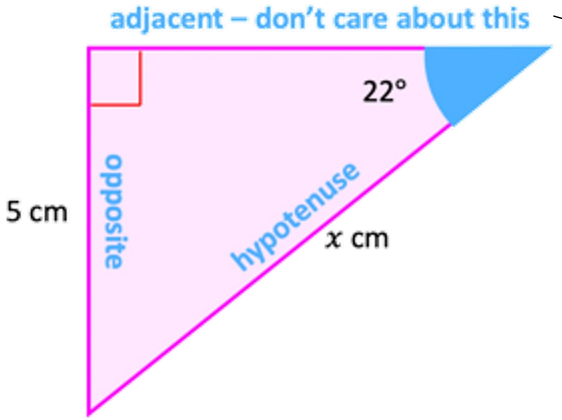
So we get $x$=14×tan34= 9.44

## Example 2:
## Finding a side using method 1 (pyramid)





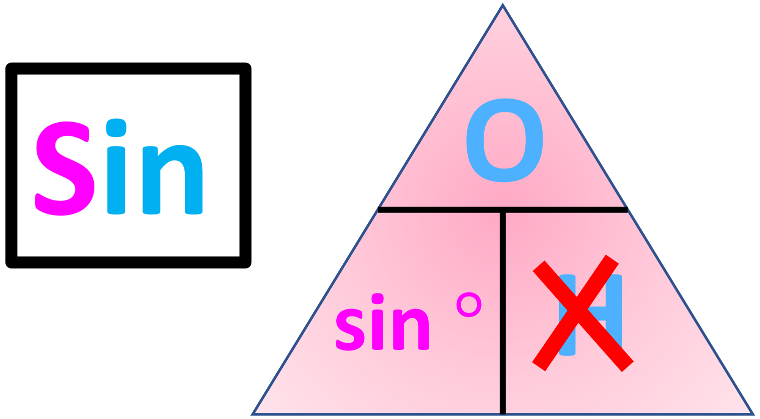

**Step 1:** Label in relation to the angle




Note: we don't care about this side since not asked to find it and not given it

**Step 2:** This involves sin since we care about opposite and hypotenuse, so let's use the Sin triangle. We are trying to find hypotenuse (H), so cover up H part.

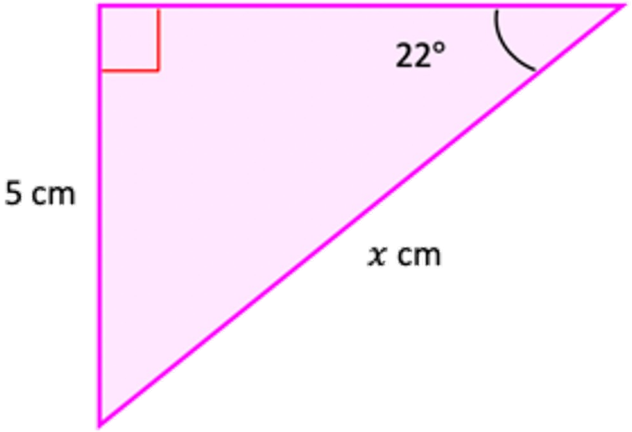
**Step 3:**



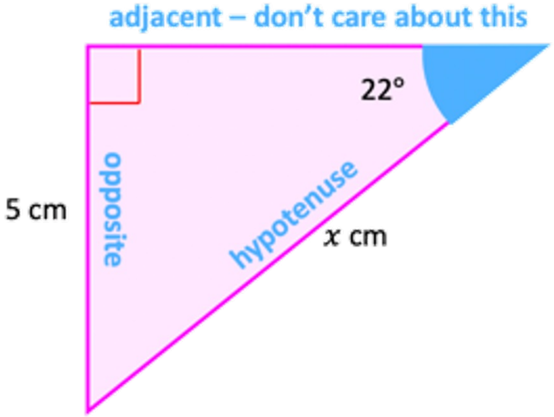

we are left with O divided by $sin(angle)$

$$\Rightarrow \frac{5}{\sin 22°} = 13.3$$

# Example 2:
# Finding a side using method 2 (algebraic)






This involves sin since we care about opposite and hypotenuse

Fill into $\sin x = \frac{opp}{hyp}$

$$\sin 22 = \frac{5}{x}$$

Here it is a bit harder to re-arrange for $x$. It always is when the $x$ is in the denominator! Forget about $\sin 22 = \frac{5}{x}$ for a minute.

If you had some other easier equation like $3 = \frac{x}{5}$, what would you do?

You'd multiply across by $x$ first to get $3x = 5$ and then you'd divide by 3

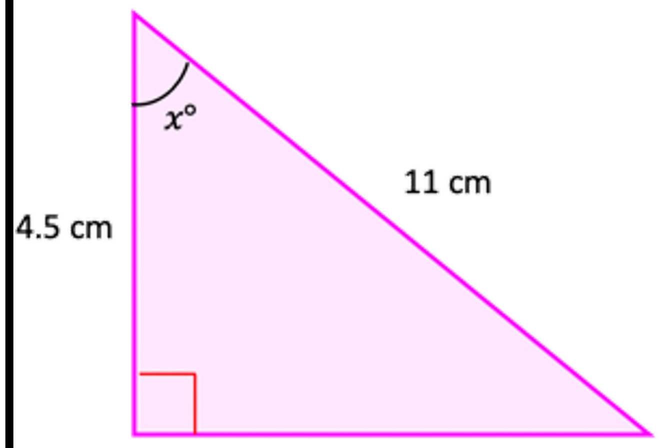
We do the exact same here with $\sin 22 = \frac{5}{x}$

First multiply $x$ across to get $x \sin 22 = 5$. This is the same as writing $\sin 22\, x = 5$

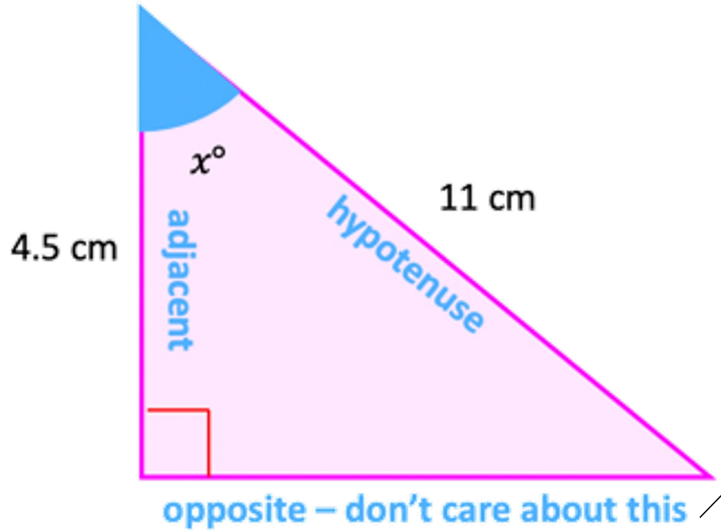
Remember $\sin 22$ is just a number, don't let it confuse you

$$x = \frac{5}{\sin 22} = 13.3$$

# Example 3:
# Finding an angle using method 1 (pyramid)



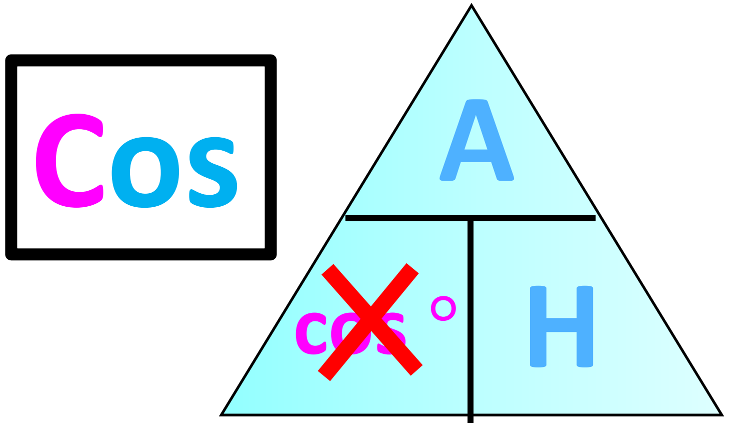

**Step 1:** Label in relation to the angle




Note: we don't care about this since not asked to find it and not given it

**Step 2:** This involves cos since we care about adjacent and hypotenuse, so let's use the Cos triangle. We are trying to find angle so cover up cos° part.

**Step 3:**



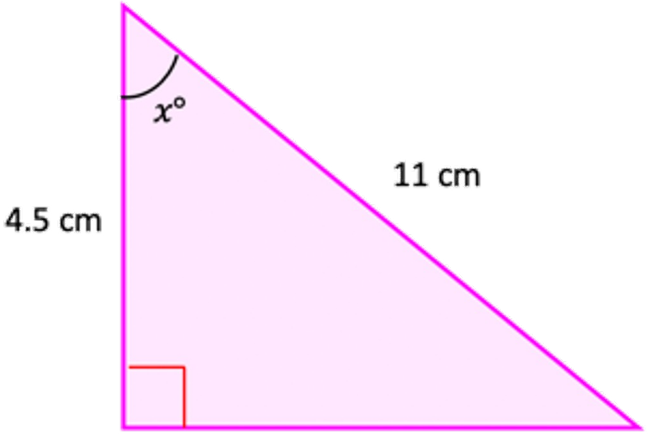

Careful: When finding an angle, we have a fraction and must use the shift button on the calculator

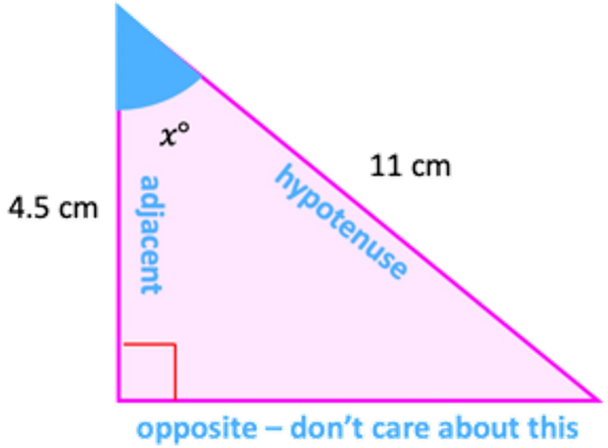
We are left with $\frac{A}{H} \Longrightarrow \frac{4.5}{22}$

**WATCH OUT**: When finding an angle we must use the trig button with the $-1$ hence $cos^{-1}$

$$\Longrightarrow cos^{-1}\left(\frac{4.5}{22}\right) = 65.8°$$

# Example 3:
# Finding an angle using method 2 (algebraic)




This involves cos since we care about adjacent and hypotenuse

Fill into $\cos x = \frac{adj}{hyp}$

$$\cos x = 4.511$$

We are finding an angle so need to use the $-1$ button

$$x = \cos^{-1}(4.511) = 65.8°$$